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Lifting Surface Approach to the Estimation of Gust Response of Finite Wings

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Unsteady response of finite wings flying though turbulence is investigated using linearized lifting surface theory. The NLR method, extended to the unsteady case, is employed for the numerical calculation of the lift force arising from oscillatory gusts for a wide range of frequencies. An efficient numerical procedure is presented for the estimation of the response to random turbulence. It takes into consideration the instantaneous variation of the vertical velocity of turbulence along the span as well as the spanwise distribution of the aerodynamic influence function. This procedure uses the relation between the output power spectral density and the spanwise cross-spectrum of turbulence, defined in terms of the spanwise correlated frequency response function, which in turn is derived from the aerodynamic transfer matrix for the modified upwash field. The results computed by means of this method with a model of turbulence uniform in span, and with a model of isotropic turbulence, exhibit considerable differences. Moreover the calculated lift spectrum is influenced by the spanwise load distribution which varies in shape according to the wing planform and the frequency of gusts.

Nomenclature

A	= aspect ratio; $2s/\bar{c}$
$\bar{c}$	= geometric mean chord
$H(\omega,y)$	= spanwise correlated aerodynamic transfer function
J_n	= Bessel function of the first kind, order n
$K_{0,1}$	= modified Bessel function of the second kind, order 0 and 1
$K(x_0,y_0;M,k)$	= kernel function in integral Eq. (1)
k	= reduced frequency; $\omega\bar{c}/2U$
L	= integral scale of turbulence
$l(x,y)$	= nondimensional lift per unit area
M	= Mach number
m	= number of spanwise collocation stations
N	= number of chordwise functions or collocation points
$S(k)$	= Sears function; $C_L(k)/2\pi$
$S'(k)$	= modified Sears function; $S(k)\exp(-ik)$
s	= semi-span of wing
$w(x,y)$	= local upwash velocity
$\tilde{w}(x,y)$	= modified upwash velocity; $w(x,y)\exp(2ikx/\bar{c})$
η	= spanwise ordinate; y/s
$\Phi(\omega)$	= one-dimensional power spectrum
$\tilde{\Phi}(\omega,\Delta y)$	= spanwise correlated power spectrum
$\tilde{\psi}(\Delta x,\Delta y)$	= two-dimensional correlation function
$\Omega_q(p,\nu,r)$	= element of aerodynamic influence matrix
ω	= circular frequency of oscillation

Superscript

$(\ )^*$	= complex conjugate

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I. Introduction

THE problem of estimating the response of a wing passing through turbulence has been given considerable attention by many researchers. The lift response of two-dimensional or large aspect ratio wings to random turbulence has been studied by Diederich[1] and Filotas,[2] taking into account the spanwise correlation of the local upwash velocities. Both of these analyses, however, have been based upon the approximation that the spanwise load distribution due to gusts has an assumed shape, i.e., a rectangular or elliptic one. Therefore, they do not take into account the variation of the spanwise loading according to the frequency of gusts or the aspect ratio of the wing.

This problem is re-investigated in the present study by the use of subsonic lifting surface theory. A powerful approach to the numerical evaluation of the power spectrum of the lift force arising from turbulence is presented, which is employed to make comparisons for several planforms of results obtained with a model of turbulence uniform in span and a model of isotropic turbulence. Jackson et al.[3] have employed lifting surface theory in the calculation of the gust response of a rectangular wing in turbulent flow. Their approach differs from the one of the present authors. Being restricted to the case of rectangular planforms, it has the advantage of avoiding the use of collocation techniques.

In the following section, we calculate the response due to sinusoidal gusts for the purpose of examining the capability of the numerical lifting surface theory employed in the present study. The lift force and load distributions of several wings are discussed.

II. Response to Sinusoidal Gust

In two-dimensional potential flow theory, the frequency response of a thin wing to a gust varying sinusoidally in the direction of flight is known as the Sears function. As for a wing with finite span, lifting line theory has been applied to the evaluation of the gust loads in the limit as the aspect ratio of the wing under consideration is very low or very high. These works, however, are rather restricted in their availability and applicability. The lift force acting on a wing with a general planform can be evaluated more appropriately by use of lifting surface theory. The unsteady version of the theory has been advanced mainly for studies of aerodynamic flutter phenomena. Many researchers have developed excellent numerical methods that yield reliable results for arbitrary planforms and for boundary conditions that vary in a complex manner, as in the case of gust problems. It may be said, especially in the unsteady case, that the theory has become practically applicable owing to the development of such refined numerical methods and the availability of large digital computers. Among these numerical methods, the one which was originally developed in the NLR[4] and later extended to unsteady case by Lehrian and Garner[5] is employed in the present study.

Extended NLR Method

Let us review briefly the numerical method used for calculating the aerodynamic loads arising from gusts.

We take the origin of the reference frame at the leading edge on the center line of the planform. Nondimensional lift distribution on a thin wing, in a flow of uniform velocity U, can be related to the local upwash angle, which is harmonically oscillating with a small amplitude, by the following integral equation.

$$\frac{w(x,y)}{U} = -\frac{1}{8\pi}\int\int l(x',y')K(x_0,y_0;M,k)\,\mathrm{d}x'\mathrm{d}y' \quad (1)$$

where $x_0 = x - x'$, $y_0 = y - y'$, and $k = \omega\bar{c}/2U$. An analytical formulation of the kernel function $K(x_0, y_0, M,k)$ has been presented by Watkins et al.[6] The spanwise integral in Eq. (1) has a singularity due to a dipole in the kernel function. Since this integral equation cannot be solved analytically for a general planform, a numerical approach is employed.

The extended NLR method used in the present calculation is one of the modal-function-type numerical methods, in which the solution of the integral equation is given by a superposition of a number of assumed distribution functions whose coefficients are determined by simultaneous algebraic equations. A distinctive feature of the NLR method is that the numerical efficiency in the evaluation of the above mentioned singular integral is greatly improved by evaluating the singular part using a Taylor expansion of the influence function followed by numerical integration of Gaussian type for the remainder part in place of direct integration of the original singular function. The method has the added advantage that a larger number of interpolation points may be taken for the spanwise integration than the number of the assumed loading functions.

The method can be improved still more following the suggestion by Ichikawa[7] and selecting the interpolation points for the spanwise integration not to coincide with those for the loading functions thereby eliminating the need for the second derivatives of the regularized influence function. This reduces the computational time greatly, especially in the unsteady case.

The nondimensional load distribution on the wing is described by a superposition of modal functions in the form

$$l(x',y') = \frac{8s}{\pi c(\eta')}\sum_{q=1}^{N}\Gamma_q(\eta')\Psi_q(\varphi')\exp\left(\frac{-ikx'}{\bar{c}/2}\right) \quad (2)$$

where the spanwise distribution $\Gamma_q(\eta)$, $q=1,2,\ldots,N$ are to be determined,

$$\Psi_q(\varphi') = \frac{\cos(q-1)\varphi' - \cos q\varphi'}{\sin\varphi'} \qquad (q=1,2,\ldots,N) \quad (3)$$

Denoting the x-ordinate of the leading edge x_l, the angular chordwise parameter φ' is given by

$$x' = x_l(\eta') - \frac{c(\eta')}{2}(1-\cos\varphi') \qquad (0 \le \varphi' \le \pi) \quad (4)$$

Defining a modified local upwash angle as

$$\frac{\tilde{w}(x,y)}{U} = \frac{w(x,y)}{U}\exp\left(\frac{ikx}{\bar{c}/2}\right) \quad (5)$$

and substituting Eq. (2) into the integral equation, Eq. (1), yields

$$\frac{\tilde{w}(x,y)}{U} = \frac{1}{2\pi}\sum_{q=1}^{N} ⨎_{-1}^{1}\frac{\Gamma_q(\eta')F_q(x,\eta,\eta')}{(\eta-\eta')^2}\,\mathrm{d}\eta' \quad (6)$$

where F_q is the influence function given by

$$F_q(x,\eta,\eta') = -\frac{1}{\pi}\int_0^{\pi}\left[y_0^2\exp\left(\frac{ikx_0}{\bar{c}/2}\right)K(x_0,y_0;M,k)\right] \times \Psi_q(\varphi')\sin\varphi'\,\mathrm{d}\varphi' \quad (7)$$

Let the coordinates of the collocation points (at which the local upwash angle is given) be (ξ_p, η_ν), where

$$\begin{cases} \xi_{p\nu} = \dfrac{x_l(\eta_\nu)}{\bar{c}} - \dfrac{c(\eta_\nu)}{2\bar{c}}(1-\cos\varphi_p), \quad \varphi_p = \dfrac{2p\pi}{2N+1} \quad (p=1,2,\ldots,N) & (8) \\ \eta_\nu = -\cos\theta_\nu, \quad \theta_\nu = \dfrac{\nu\pi}{m+1} \quad (\nu=1,2,\ldots,m) & (9) \end{cases}$$

and interpolate the spanwise distribution function Γ_q at m spanwise collocation points $\eta_r = -\cos\theta_r$, $r=1,2,\ldots,m$ by the following double Fourier series:

$$\Gamma_q(\eta') = \frac{2}{m+1}\sum_{r=1}^{m}\Gamma_{qr}\sum_{\mu=1}^{m}\sin\mu\theta'\,\sin\mu\theta_r \quad (q=1,2,\ldots,N) \quad (10)$$

As a result the integral equation is transformed into the following simultaneous algebraic equations with $N\times m$ unknowns.

$$\sum_{q=1}^{N}\sum_{r=1}^{m}\Omega_q(p,\nu,r)\Gamma_{qr} = \frac{\tilde{w}(\xi_{p\nu},\eta_\nu)}{U} \quad (p=1,\ldots,N;\ \nu=1,\ldots,m) \quad (11)$$

In the determination of the coefficients $\Omega_q(p,\nu,r)$ the needed evaluation of singular integrals is made following the procedure described earlier.

In the gust response analysis an upwash varying sinusoidally along the chord is considered, $w(x,y) = \exp(-i\omega x/U)$, making the modified upwash angle given by Eq. (5) equal to unity at every collocation point. After solving the simultaneous equations for Γ_{qr} the nondimensional load distribution is obtained from Eqs. (2), (3) and (10), and integration over the wing area gives the lift force in nondimensional form as follows:

$$C_L(k) = \frac{2s}{\bar{c}(m+1)}\sum_{q=1}^{N}\sum_{r=1}^{m}\Gamma_{qr}K_q(\eta_r)\sin\theta_r \quad (12)$$

where

$$K_q(\eta) = \exp\left(\frac{-ik(x_l - c/2)}{\bar{c}/2}\right)\left[i^{q-1}J_{q-1}\left(\frac{kc}{\bar{c}}\right) - i^q J_q\left(\frac{kc}{\bar{c}}\right)\right] \quad (13)$$

Some Results of the Calculation

The numerical approach described previously has been employed in the estimation of the lift response to sinusoidal gusts of finite wings and with various planforms. The present method is found to yield a satisfactory result for every planform and to cover a wide range of frequencies with acceptable accuracy. Some of the results are discussed briefly in the following.

For the rectangular planforms investigated here at moderate reduced frequencies ($k<1.0$), 33 collocation points ($N=3$, $m=11$) were used and 47 spanwise interpolation points for the regularized influence function were selected giving the solution to three decimals. The response to unit upwash was also computed to check the accuracy of the result following Ref. 5. All the computations were carried out by the Data Processing Center, Kyoto University, on the FACOM M-190 Computer. Running time required for each case was about 20 s.

The gust response function, defined as $S(k)=C_L(k)/2\pi$, has been computed for rectangular planforms and is illustrated in Fig. 1. The results of the present calculation are represented by solid curves, with the exception that the top curve, labelled $A=\infty$, is the Sears function in two-dimensional theory. The curve for the case $A=6$ is in good agreement with the earlier result of lifting surface theory,[8] computed by means of a different numerical scheme. Previous results of lifting line theory[9,10] calculated with the assumption of very high, or very low, aspect ratio are also plotted in the same figure. The result obtained with the low aspect ratio approximation for the case $A=2$ agrees well with the present result except for a slight difference at higher frequencies. However, the high aspect ratio approximation results in an overestimation of the response even for the case $A=6$.

It should be noted that the reference point for the sinusoidal wave of upwash, i.e., the origin of the coordinates, is at the leading edge in the present calculation, whereas it is at midchord for the two-dimensional Sears function. The original result of the present calculation may be referred to as the modified Sears function,[11] $S'(k)$, from which the usual gust response function is derived by the relation $S'(k)=S(k)\exp(-ik)$. In Fig. 2, the modified Sears function is plotted on the complex plane with the reduced frequency as a parameter. The results agree with those in the recent paper by Jordan,[12] in which a refined doublet lattice type method is developed for rectangular planforms. The present figure suggests that as k goes to infinity, the phase angle reaches the limit $-\pi/4$ the same as in the two-dimensional case. Figure 3 is an illustration of the spanwise distribution of the response at various frequencies. It shows that the distribution clearly has an elliptic shape when the frequency is low, but tends to become uniform along the span as the frequency increases.

It can be surmised from the above results that, as long as linearized potential flow theory is applicable, the response function of a finite wing to sinusoidal gusts approaches the two-dimensional Sears function as the reduced frequency becomes very high; in other words, the effect of the wing-tip decreases with increasing frequency.

The response function of a tapered swept wing (aspect ratio is 6, angle of sweepback of midchord is 30 deg and taper ratio is ⅓) is plotted in Fig. 4 for several subsonic Mach numbers. The figure shows that the response is more sensitive to the change of the reduced frequency at the higher Mach numbers. The spanwise load distribution for the case $M=0$ is shown in Fig. 5. When the frequency is high, the distribution is affected strongly by the phase shift due to sweepback angle.

III. Response to Random Gusts

One approach for analyzing the response of an aerofoil passing through turbulence is to assume that the local velocity of the vertical gust fluctuates along the chord, but remains

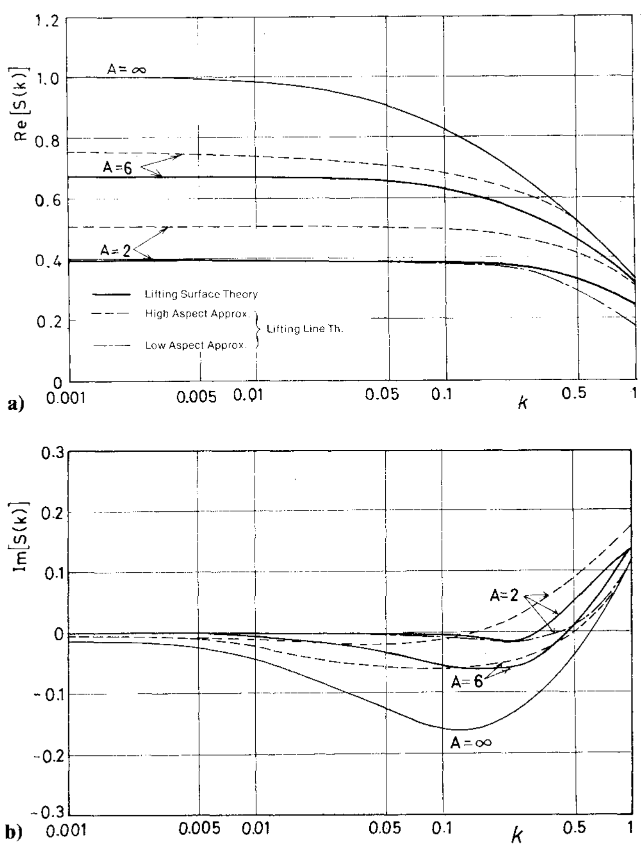


Fig. 1 Gust response function of rectangular wings ($M=0$). a) real part; b) imaginary part.

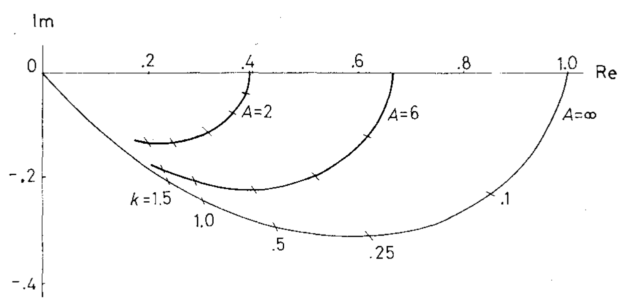


Fig. 2 Modified Sears function $S'(k)=S(k)e^{-ik}$.

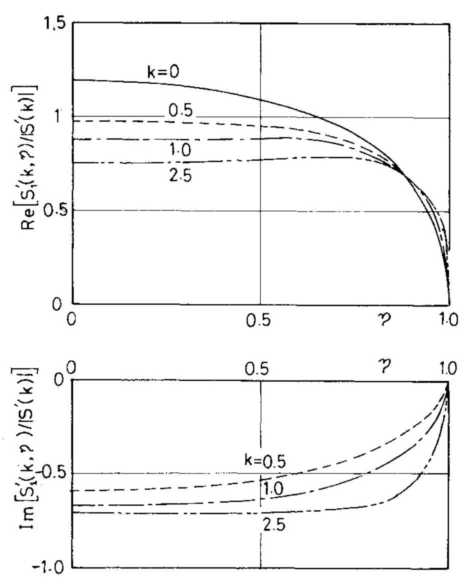


Fig. 3 Spanwise load distribution (rectangular wing, $A=6$).

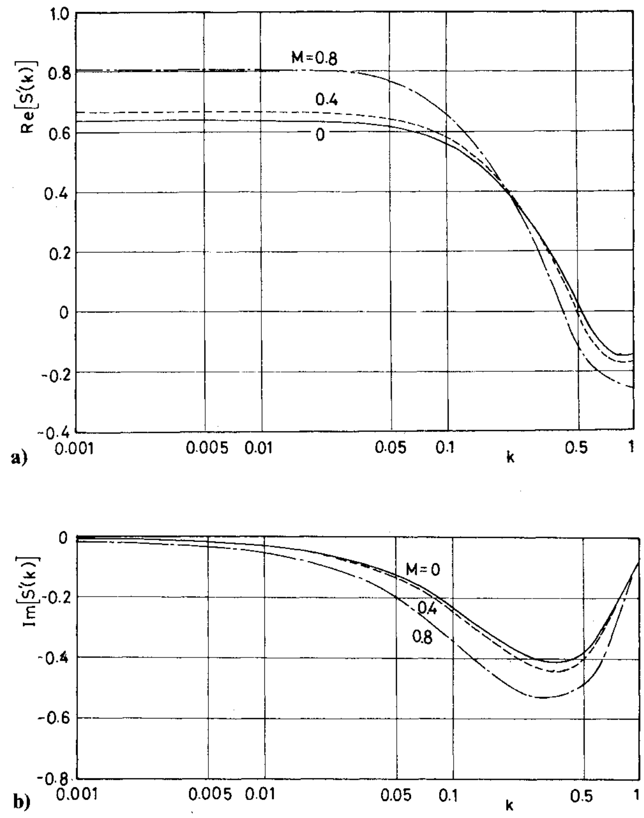


Fig. 4 Gust response function of a tapered swept wing.

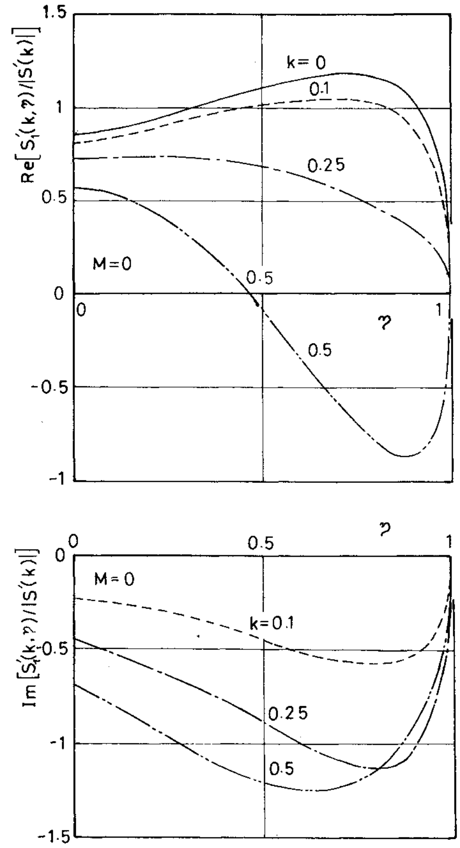


Fig. 5 Spanwise load distribution (tapered swept wing).

constant along the span, as is illustrated in Fig. 6a. This model of turbulence is referred to as being "one-dimensional," and is often employed in practical use due to its simplicity. The power spectral density (PSD) of the lift force can then easily be evaluated under the assumption of stationary homogeneous turbulence, using the techniques of generalized harmonic analysis. It is written in the form

$$\Phi_{C_L}(\omega)=\left(\frac{4\pi^2}{U^2}\right)|S(k)|^2\Phi_w(\omega) \qquad (14)$$

where Φ_w is the one-dimensional PSD of turbulence for which Dryden's well-known model of isotropic turbulence usually is adopted.

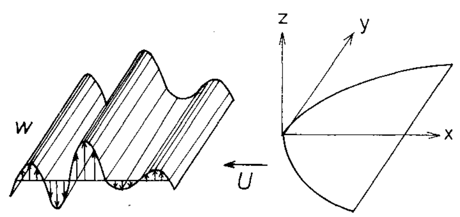
The more complicated analysis based on the "two-dimensional" model of turbulence is, however, required when the representative wavelength of turbulence is not very large compared with the wing span, and the instantaneous variation of the gust velocity along the span no longer is negligible (Fig. 6b). Diederich[1] has investigated the response of an aerofoil in such two-dimensional turbulence. He used the relation between the lift force spectrum and the two-dimensional turbulence spectrum defined in terms of the spanwise correlated frequency response function. It can be written in the form

$$\Phi_{C_L}(\omega)=\frac{1}{U^2}\int_{-s}^{s}\int_{-s}^{s}\{H(\omega,y)\}^{*}H(\omega,y')\tilde{\Phi}_w(\omega,y'-y)\,dy\,dy' \qquad (15)$$

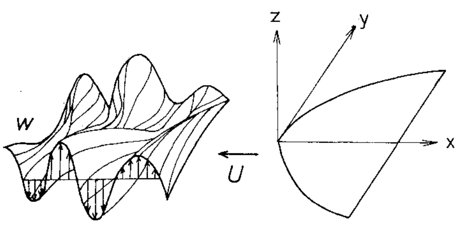
where $\tilde{\Phi}_w$ is the spanwise correlated turbulence spectrum defined as

$$\tilde{\Phi}_w(\omega,\Delta y)=\frac{1}{\pi}\int_{-\infty}^{\infty}\exp(-i\omega\tau)\tilde{\psi}_w(U\tau,\Delta y)\,d\tau \qquad (16)$$

which is the single Fourier transform of the two-dimensional correlation function. If Dryden's model of turbulence



a) One-dimensional turbulence



b) Two-dimensional turbulence

Fig. 6 Illustration of the turbulent field; a) spanwise uniform, and b) isotropic.

spectrum is employed, one obtains

$$\tilde{\Phi}_w(\omega,\Delta y)=\frac{L\overline{w^2}}{\pi U}\frac{1}{(1+k_L^2)^2}\left[(1+3k_L^2)\zeta K_1(\zeta)-\zeta^2K_0(\zeta)\right] \qquad (17)$$

where $\zeta=(|\Delta y|/L)\sqrt{1+k_L^2}$, $k_L=\omega L/U$, Φ_w is the one-dimensional spectrum of turbulence given by

$$\Phi_w(\omega)=\frac{L\overline{w^2}}{\pi U}\frac{1+3\omega^2L^2/U^2}{(1+\omega^2L^2/U^2)^2} \qquad (18)$$

The spanwise correlated frequency response function $H(\omega,y)$ can be interpreted as the response to the incidence which is a sinusoidal wave along the chord and a spanwise impulse function. Therefore we can use the aerodynamic transfer matrix derived for the calculation of the response to sinusoidal gusts. Numerical evaluation of the lift spectrum by use of Eq. (15) is straightforward. The integrals are evaluated using the same interpolation as before:

$$\Phi_{C_L}(\omega)=\frac{1}{U^2}\sum_{\nu=1}^{m}\sum_{\nu'=1}^{m}\{G(\omega,\nu)\}^{*}G(\omega,\nu')\tilde{\Phi}_w(\omega,\eta_{\nu'}-\eta_\nu) \tag{19}$$

where

$$G(\omega,\nu)=\frac{2s}{\bar{c}(m+1)}\sum_{p=1}^{N}\sum_{q=1}^{N}\sum_{r=1}^{m}\Omega_q^{-1}(p,\nu,r)K_q(\eta_r)\sin\theta_r \tag{20}$$

and $\Omega_q^{-1}(p,\nu,r)$ is an element of the inverse of the aerodynamic influence matrix in Eq. (11).

A formula similar to the above expression has been proposed by Coupry.[13] His approach leads to the square matrix representation of the pressure field at a number of relative locations on the wing (after manipulating the aerodynamic influence matrix and the square matrix representing the cross power of the turbulence field, evaluated for the relative positions). This expression, however, explicitly includes the terms of the chordwise phase angle which varies with each set of relative positions. Therefore, the higher the frequency becomes, the more the number of calculating points along the chord must be increased to give an adequate description of the variation in phase angle for accurate integration of the chordwise correlated pressure field. This situation is not desirable, especially not if the use of a mode-function type of numerical method is intended. This problem is alleviated in the present approach by employing the spanwise correlated frequency response function.

The lift spectrum has been computed for a gust field that is sinusoidal along the chord and has two different types of spanwise distributions, isotropic and uniform. It is plotted in Fig. 7 for the case of a rectangular wing of aspect ratio 6, with the semispan ratio to the scale of turbulence as a parameter. The figure shows that use of the isotropic turbulence model gives lower values than use of the uniform turbulence model. Figure 8 shows the admittance, derived as the ratio between lift and turbulence velocity spectra, plotted for the case $s/L=1.0$. There are some discrepancies between the results computed for the different planforms.

These results give evidence that the assumption of turbulence uniform in span is no longer valid if the representative wavelength of turbulence is not very large in comparison to the wing span. A more complicated approach, as has been proposed in this section, is required for adequate evaluation of the gust loads arising from turbulence with such a small wavelength.

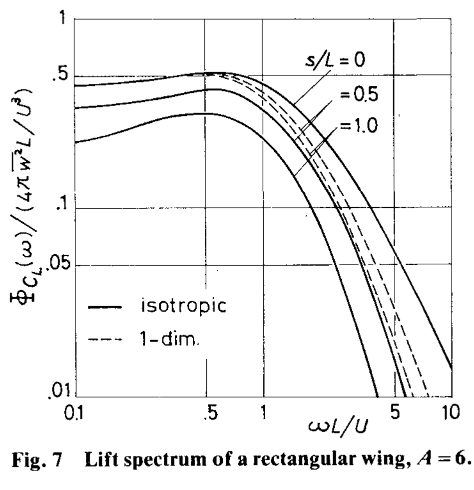


Fig. 7 Lift spectrum of a rectangular wing, $A=6$.

An approximate expression of the lift spectrum, derived analytically by using Dryden's spectrum, has been given by Diederich. He incorporated the two-dimensional model of turbulence into his analysis with the assumption that the spanwise distribution of the correlated aerodynamic transfer function $H(\omega,y)$ is independent of the frequency and remains constant along the span.

Another approximate expression has been proposed by Filotas.[2] He derived it through a different approach by employing the two wavenumber transfer function which gives the lift response of a wing flying through a yawed sinusoidal gust of arbitrary orientation and wavelength. He also assumed an elliptic shape for the spanwise distribution of the gust loads.

The results of these approximations metioned above are illustrated in Fig. 9 together with that of the present calculation. It should be noted that the ordinate is the ratio between the response computed with the model of two-dimensional isotropic gust field and that computed with the model of turbulence uniform in span. In this manner the discrepancies between the results are shown more clearly. In the case of rectangular planforms, the values computed with the present method fall between those computed by the two approximate methods. At low frequencies the present result is close to the approximate value of Filotas, whereas it approaches the value of Diederich as the frequency parameter becomes large. This is what may be expected from the result described in the preceding section, i.e., the spanwise load distribution computed for a rectangular wing has the tendency of becoming constant along the span as the frequency increases. Therefore, if the frequency parameter k_L is large, as

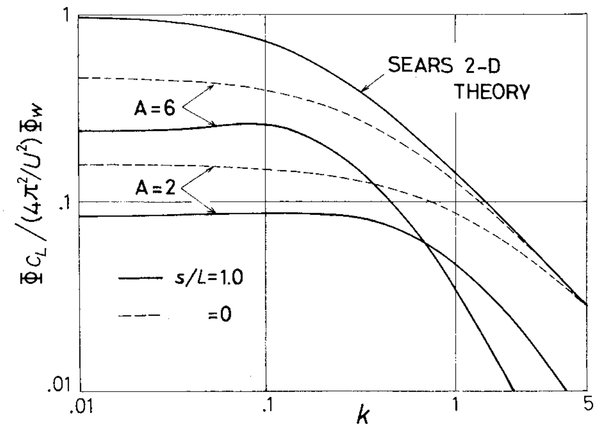


Fig. 8 Admittance of rectangular wings.

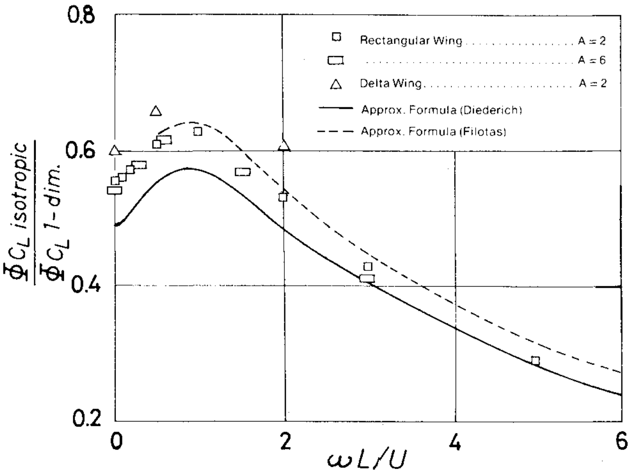


Fig. 9 Ratio of two-dimensional to one-dimensional output response.

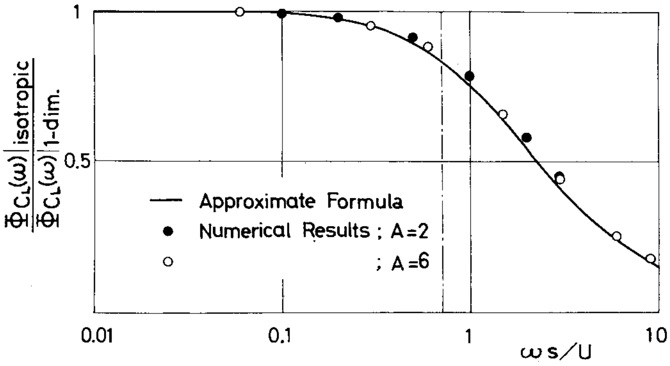


Fig. 10 Approximate formula giving the criterion of the validity of one-dimensional analysis.

is usually the case with an airplane in atmospheric turbulence, and the planform of the wing is nearly rectangular, the power spectral density of the lift force arising from turbulence may adequately be evaluated by following the approximate formula of Diederich. Furthermore, in this range of k_L, his formula can be modified into a simpler form as

$$\frac{\Phi_{C_L}(\omega)\text{ isotropic}}{\Phi_{C_L}(\omega)\,1-\text{dim.}} = \frac{1}{k_s^2}\left[k_s Ki_0(2k_s) + 2k_s K_1(2k_s) - 1\right] \tag{21}$$

where $k_s = \omega s/U$ and

$$Ki_0(x) = \int_0^x K_0(\xi)\,\mathrm{d}\xi \tag{22}$$

It is plotted in Fig. 10 against the parameter k_s together with some results of the present lifting surface calculation for the case of rectangular planforms. Coupry[13] has also presented a criterion for the validity of the assumption of turbulence uniform in span when calculating the lift spectrum. He has shown that the computation must take into account the spanwise correlation of the upwash field when the ratio of the transverse coherence length to the span dimension is of order unity or less. This is equivalent to saying that the one-dimensional model of turbulence is valid for the frequency parameter $k_s = \omega_s/U < 0.715$. This critical value is indicated in Fig. 10 by a dash-dot line.

IV. Conclusions

In the previous sections a numerical approach of linearized lifting surface theory, i.e., the NLR method extended to unsteady case, has been applied to the estimation of the lift force generated on a finite wing in turbulent flow. The power spectral density of the lift force has been computed by an efficient numerical procedure which employs the spanwise correlated aerodynamic transfer functions, derived from the frequency response to the modified upwash field.

The comparisons of the lift spectra computed with a model of turbulence uniform in span and with a model of isotropic turbulence exhibit considerable discrepancies. When the semispan ratio to the scale of turbulence approaches unity the assumption of turbulence uniform in span is no longer valid and results in an overestimate of the gust load.

It also has been found that the variation of the spanwise gust-load distribution affects distinctively the resulting lift spectra. The value of the lift spectrum computed for rectangular planform has been shown to be bounded by two approximate theories. At low frequencies it is in good agreement with the approximate value derived by Filotas[2] with the assumption of elliptic spanwise load distribution, whereas it approaches the value obtained by Diederich[1] with a uniform load distribution as the frequency increases. As has been mentioned earlier this result may be accounted for by the fact that the spanwise lift distribution has a tendency to become uniform with increasing frequency for a spanwise uniform downwash.

Finally it should be pointed out that an essential assumption has been made throughout the present study: it has been assumed that the loads arising from gusts can be estimated using potential flow theory, even though turbulence, by its nature, is a nonpotential flow. This assumption may be justified in the case of gusts with large wavelengths in comparison to the wing chord, but for gusts of much smaller wavelengths, which are closely related to excitation of the flexible airplane modes the gust can no longer be regarded as potential flow. More experimental research is required to clarify how far potential theory can be used to estimate gust loads.

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